Large two-loop contributions to $g-2$ from a generic pseudoscalar boson

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We calculate the dominant contributions to the muon $g-2$ at the two-loop level due to a generic pseudoscalar boson that may exist in any exotic Higgs sector in most extensions of the standard model. The leading effect comes from diagrams of the Barr-Zee type. A sufficiently light pseudoscalar Higgs boson can give rise to contributions as large as the electroweak contribution which is measurable in the next round of $g-2$ experiments. Through the contribution we calculate here, the anticipated improved data in the recent future on the muon $g-2$ can put the best limit on the possible existence of a light pseudoscalar boson in physics beyond the standard model.

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Precision measurement of the anomalous magnetic moment of the muon, $a_\mu \equiv \frac{1}{2}(g_\mu - 2)$, can provide not only a sensitive test of quantum loop effects in the electroweak standard model (SM), but can also probe the effects of some potential ''new physics.'' The experimental average of the 1998 Particle Data Group gives [1] $a_\mu^{\rm exp} = 11659230(84) \times 10^{-10}(\pm 7.2\ \text{ppm})$. Recent measurements by the E821 experiment at Brookhaven gives [2] $a_\mu^{\rm exp} = 11659250(150) \times 10^{-10}(\pm 13\ \text{ppm})$ (1997 data) and $a_\mu^{\rm exp} = 11659191(59) \times 10^{-10}$ (1998 data). Combining with previous data, this can be translated [3] into

$$a_\mu^{\rm exp} = 11659210(46) \times 10^{-10}(\pm 3.9\ \text{ppm}). \tag{1}$$

The E821 experiment is expected [4] to soon reduce the error by more than a factor of 10 to ± 0.35 ppm [3] with data from one month of dedicated running. With subsequent longer dedicated runs, it could statistically approach the anticipated systematic uncertainty of about $\pm(10-20)\times 10^{-11}$ [5].

The contributions to a_μ are traditionally divided into

$$a_\mu = a_\mu^{\rm QED} + a_\mu^{\rm Hadronic} + a_\mu^{\rm EW} + \Delta a_\mu, \tag{2}$$

representing QED, hadronic, electroweak, and the exotic (beyond the standard model) contributions, respectively. The QED loop contributions have been computed to very high order [6]

$$a_\mu^{\rm QED} = \frac{\alpha}{2\pi} + 0.765857381(51)\left(\frac{\alpha}{\pi}\right)^2 + 24.050531(40)\left(\frac{\alpha}{\pi}\right)^3 + 126.02(42)\left(\frac{\alpha}{\pi}\right)^4 + 930(170)\left(\frac{\alpha}{\pi}\right)^5. \tag{3}$$

The most precise value for the fine structure constant $\alpha = 1/137.03599944(57)$ can be obtained by inverting the similar formula for the electron $g_e - 2$ from the data [7]. This gives

$$a_\mu^{\rm QED} = 116584706(2) \times 10^{-11}, \tag{4}$$

much more precise than the expected experimental reach. The hadronic contribution due to the hadronic vacuum polarization diagram is $a_\mu^{\rm Hadronic} = 6771(77)\times 10^{-11}$ [8]. The SM electroweak contribution up to the two-loop level gives $a_\mu^{\rm EW} = 151(4)\times 10^{-11}$ [6,9] for $\sin^2\theta_W = 0.224$ and $M_H = 250$ GeV (in comparison, the one-loop SM electroweak contribution is 195×10^{-11}). The total value in the standard model is

$$a_\mu(\text{SM}) = a_\mu^{\rm QED} + a_\mu^{\rm Hadronic} + a_\mu^{\rm EW} = 116591628(77)\times 10^{-11}(\pm 0.66\ \text{ppm}). \tag{5}$$

The biggest theoretical uncertainty still comes from the strong interaction; however, it is still smaller than the experimental uncertainty. The hadronic uncertainty can be reduced further by measuring the hadronic vacuum polarization effect directly, and there are many experiments which intend to achieve this goal.

Compared with the latest experimental value, the two are still consistent. However, one can tell that the experimental value is biased toward the high side of the standard model prediction. Given that $a_\mu^{\rm Hadronic}$ and $a_\mu^{\rm EW,one\text{-}loop}$ are both positive, one can conclude that the current data already probe these contributions. Note that $a_\mu^{\rm EW,two\text{-}loop}$ is negative. Naively one can extract from the SM prediction and data that the new physics contribution, Δa_μ, between $(-31.0-+121.6)\times 10^{-10}$ is still allowed at (one-sided) 95% C.L. [10]. It will be very interesting to see if there is disagreement if the experimental data is reduced by a factor of 10 as expected.

Even without the recent experimental improvement, $g-2$ data has already provided nontrivial constraints [11] on physics beyond the standard model. For example, the constraint on the minimal supersymmetric standard model (MSSM) due to its one-loop contribution to $g-2$, via smuon-smuon-neutralino and chargino-chargino-sneutrino loops, is well known [12]. The resulting constraint depends on the masses of supersymmetric particles and $\tan\beta$.

In theories beyond the standard model, there are usually additional scalar or pseudoscalar bosons. In particular, some of the pseudoscalar bosons can potentially be light because

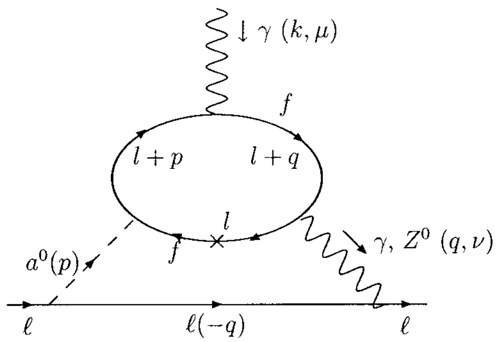

FIG. 1. The dominant two-loop graph involving a pseudoscalar boson that contributes to g_l-2. The cross location denotes a possible mass insertion.

of its pseudo-Goldstone nature, accidentally or otherwise. However, in a collider search, it is known that searching for the pseudoscalar neutral boson is much harder than the scalar neutral or charged one. Therefore it is particularly interesting to see if one can constrain the pseudoscalar boson using low energy precision experiments. In this paper we wish to report that if the extended theory has a light enough pseudoscalar boson, its two-loop contribution to muon $g-2$ can be as large as the one-loop electroweak affect. As a result the muon $g-2$ can provide a very strong probe on a large class of theories beyond the standard model.

The one-loop contributions to $g-2$ from a scalar or pseudoscalar boson have been presented many times in the literature [13]. In addition to two powers of m_μ that are required by kinematics and definitions, the one-loop contribution is further suppressed by another two powers of m_μ/M_a. However, the result is enhanced by a logarithmic loop factor, $\ln m_\mu/M_a$, coming from the diagram in which the photon is emitted by the internal muon. Therefore, for a light enough Higgs mass, some limit can be derived from $g-2$ data just based on one-loop results. Nevertheless, as we shall see later, the two-loop contribution is typically larger than the one-loop one by a factor of 2–10 for a Higgs boson mass from 10–100 GeV. In addition, the one-loop and two-loop contributions have different signs for both scalar and pseudoscalar contributions. Therefore the one-loop contribution actually partially cancels the larger two-loop contribution.

The two-loop contribution of a scalar boson has been calculated in Refs. [6,9] in the context of the standard model. The contribution of any scalar boson beyond the standard model can in principle be extracted from the calculation, and we shall not dwell on this here except to note that the scalar boson gives a negative contribution while the pseudoscalar gives a positive contribution to Δa_μ. Also, we have parametrized our input Lagrangian as model independent as possible in order to make our gauge invariant result widely applicable to a large class of models.

For Higgs masses larger than roughly 3 GeV, the dominant Higgs related contribution to the muon anomalous magnetic moment is through the two-loop Barr-Zee type diagram [14], as in Fig. 1. Compared with the one-loop graph, the Yukawa coupling of the heavy fermion f in the inner loop together with the mass insertion of the heavy fermion in the two-loop graph will give rise to $(m_f/m_\mu)^2$ enhancement, which can overcome the extra loop suppression factor of $\alpha/16\pi^2$. The internal gauge boson can be a photon or a Z^0. The Z^0 contribution is typically smaller by two orders of magnitude. It is included in this Rapid Communication just for completeness. Note that unless CP violation occurs in the Higgs potential, there is no two-loop Barr-Zee type contribution to $g_\mu-2$ associated with pseudoscalar bosons and an inner gauge boson loop. The form of the gauge invariant vertex function $\Gamma^{\mu\nu}$ of a pseudoscalar boson a^0 of momentum (p) turning into two photons $(-k,\mu)$, (q,ν) due to the internal fermion or gauge boson loop is

$$\Gamma^{\mu\nu}=P(q^2)\epsilon^{\mu\nu\alpha\beta}p_\alpha q_\beta. \tag{6}$$

In general, the heavy fermion generation dominates in the loop. The Yukawa coupling is parametrized in a model independent expression

$$\mathcal{L}=i\frac{gA_f m_f}{2M_W}\bar{f}\gamma_5 f a^0. \tag{7}$$

Integrating the fermion loop momentum, we obtain the form factor

$$P(q^2)=N_c^f\frac{gA_f e^2 q_f^2 m_f^2}{8\pi^2 M_W}\int_0^1\frac{dz}{m_f^2-z(1-1)q^2}, \tag{8}$$

where m_f and q_f are the mass and the charge of the internal fermion in the loop. The color trace gives $N_c^b=N_c^t=3$, $N_c^\tau=1$. The above vertex is further connected to the lepton propagator to produce the anomalous magnetic dipole moment $a_l^{\gamma a^0}$ for the lepton l,

$$a_l^{\gamma a^0}=\frac{\alpha^2}{8\pi^2\sin^2\theta_W}\frac{m_l^2 A_l}{M_W^2}\sum_{f=t,b,\tau}N_c^f q_f^2 A_f\frac{m_f^2}{M_a^2}\mathcal{F}\left(\frac{m_f^2}{M_a^2}\right), \tag{9}$$

$$\mathcal{F}(x)=\int_0^1\frac{\ln\dfrac{x}{z(1-z)}dz}{x-z(1-z)}. \tag{10}$$

$\mathcal{F}(1)=(4/\sqrt{3})\mathrm{Cl}_2(\pi/3)$, with the Clausen's function $\mathrm{Cl}_2(\theta)=-\int_0^\theta\ln(2\sin(\theta/2))d\theta$. As $x\gg1$, $x\mathcal{F}(x)$ has the asymptotic form $2+\ln x$. On the other extreme limit $x\ll1$, $\mathcal{F}(x)$ approaches $\pi^2/3+\ln^2 x$. Our result is consistent with that from an unphysical Higgs boson in the SM [9].

For the graph with the inner photon replaced by Z^0 boson, its contribution to a_μ can be calculated in a similar fashion:

$$a_l^{Z^0a^0}=\frac{\alpha^2 m_l^2 A_l g_V^l}{8\pi^2\sin^4\theta_W\cos^4\theta_W M_Z^2}\sum_{f=t,b,\tau}\frac{N_c^f A_f q_f g_V^f m_f^2}{M_Z^2-M_a^2}\times\left[\mathcal{F}\left(\frac{m_f^2}{M_Z^2}\right)-\mathcal{F}\left(\frac{m_f^2}{M_a^2}\right)\right], \tag{11}$$

with $g_V^f=\frac{1}{2}T_3(f_L)-q_f\sin^2\theta_W$. Note that, for both pseudoscalar and scalar boson contributions, only the vector coupling of Z^0 to a heavy fermion contributes to the effect vertex due to the Furry theorem. Numerically, this Z^0 mediated

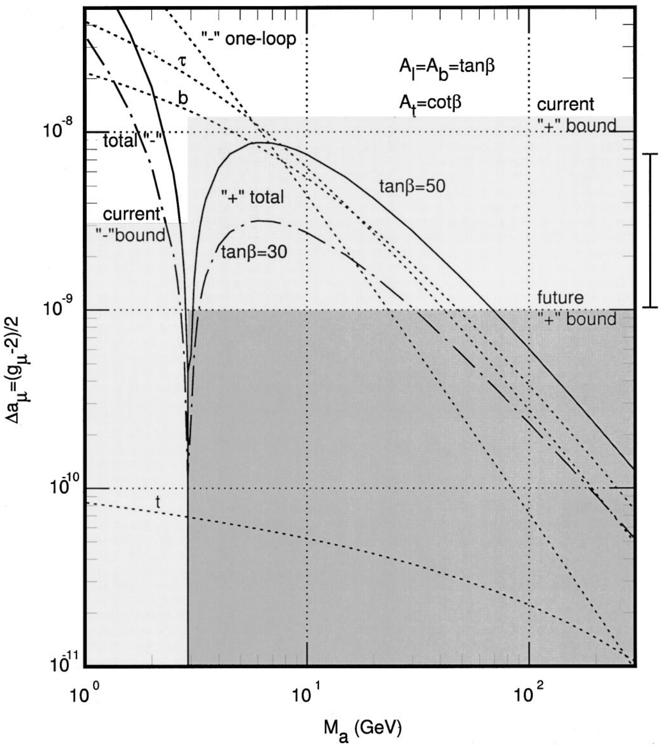

FIG. 2. Positive two-loop contributions from the inner t, b, and τ loops to $g_\mu-2$ due to the pseudoscalar a^0 versus M_a at $\tan\beta=50$, as well as the negative contribution from the one-loop diagram. The sum in the solid (dashed-dotted line) curve shows cancellation at low M_a mass for $\tan\beta=50$ (30). The lighter shaded areas are allowed by the current positive and negative bounds on the right and left sides of the zero dip of cancellation. The expected future positive bound outlines the darker shaded region.

contribution turns out to be about two orders of magnitude smaller than that of the photon mediated one. One suppression factor comes from the massive Z^0 propagator and the other one comes from the smallness of the leptonic vector coupling of the Z_0 boson, which is proportional to $(-\frac{1}{4}+\sin^2\theta_W)\sim-0.02$.

Taking the pattern of Yukawa couplings in MSSM as an example, we set A_f as $\cot\beta$ $(\tan\beta)$ for the u (or d)-type fermion. The contributions due to top quark t, bottom quark b, and tau lepton τ in the loop, respectively, as well as the total, are displayed in Fig. 2 for both $\tan\beta=30$ and 50. In this MSSM pattern the t contribution is insensitive to $\tan\beta$. In addition, both the b and τ contributions, which are roughly the same order of magnitude, dominate over that of the top quark one for large enough $\tan\beta$ and light enough pseudoscalar mass M_a. For $M_a\lesssim 15$ GeV, the τ contribution is larger than the b-quark contribution. The total two-loop photonic contribution from the pseudoscalar boson, $a_\mu^{\mu a^0}$, can be as large as 10^{-8} for a large $\tan\beta$ when $M_a\leqslant 10$ GeV, as shown in Fig. 2. For example, for $M_a=10$ GeV and $\tan\beta=50$, $a_\mu^{\gamma a^0}$ (2-loop) $=1.2\times10^{-8}$, which is above the upper limit allowed by the current experiment bound. Generically, for $M_a\sim 80-100$ GeV, $\tan\beta\sim 50$, a_μ ranges in $(7-9)\times10^{-10}$, which is close to the electroweak contribution. Note that the pseudoscalar contribution has the same sign as the hadronic or electroweak contributions.

To derive constraint from the data one must combine the one- and two-loop contributions. The well-known one-loop contribution [13] due to the pseudoscalar a^0 is

$$a_l^a(1\text{-loop})=-\frac{m_l{}^2}{8\pi^2 M_a{}^2}\left(\frac{gA_l m_l}{2M_W}\right)^2 H\left(\frac{m_l^2}{M_a^2}\right),$$

$$\text{with}\quad H(y)=\int_0^1\frac{x^3dx}{1-x+x^2y}. \tag{12}$$

For small y, $H(y)\to-\ln y-\frac{11}{6}>0$. Note that the one-loop contribution is always negative in contrast to the two-loop contribution. In Fig. 2, we draw the absolute value of the one-loop contribution for easy comparison. For small M_a such as 10 GeV, the one-loop contribution can be as large as half of the two-loop contribution and produce a canceling effect in $g_\mu-2$. Complete cancellation occurs around 3 GeV for large $\tan\beta$. However the one-loop effect becomes smaller for larger M_a due to its additional suppression factor of $(m_\mu^2/M_a^2)\ln(M_a^2/m_\mu^2)$, and is basically negligible for $M_a>50$ GeV.

To compare our results with the recent data, we note that, in the framework of the standard model, roughly an uncertainty of Δa_μ between $(-31.0-+121.6)\times10^{-10}$ can still be accommodated by the data. The lower and the upper bounds are illustrated in the shaded regions in Fig. 2. Note that for M_a lighter than roughly 3 GeV, the negative one-loop contribution dominates and the total pseudoscalar Higgs contribution becomes negative. As emphasized by previous one-loop analysis, the region with M_a lighter than roughly 2.8 GeV is already ruled out by the current $g-2$ experiment. The E821 experiment is expected [3] to announce its new result with error reduced by more than a factor of 10 very soon. It is hard to predict the consequence of this improved data since even the central value may be shifted. However, as a reference point, we plot the line $\Delta a_\mu\leqslant 10^{-9}$ in Fig. 2 as a potential consequence assuming the central value remains the same.

In CP conserving MSSM, there is a lower bound [15,16] on $M_a\geqslant 88$ GeV, which is only based on partial scanning with certain choices of benchmarks in the MSSM. Furthermore, in more general supersymmetric models or in general two or more Higgs doublet models [17], very little can be said about the potentially light pseudoscalar Higgs boson. Experimental constraint [18] on M_a from LEP data is correlated to a rather light scalar Higgs boson. The model independent nature of our calculation makes it possible to derive relatively strong limits on the pseudoscalar boson sector in any theory beyond the standard model using the hard earned data on muon $g-2$. If future data reduce the uncertainty of $g_\mu-2$ in the way that we expect, the pseudoscalar boson of less than 75 GeV for $\tan\beta=50$ can be easily ruled out except in a very narrow region of cancellation (around 3 GeV).

Note that in general multi-Higgs doublet models, the $\tan\beta$ factor in our analysis may be supplemented by additional factors of mixing matrix elements. In addition, in any specific model, there may be additional two-loop contributions, such as the ones involving the physical charged Higgs boson or the neutral scalar boson. We assume that the these contributions do not accidentally cancel each other. Given that the experimental limit on the masses of the charged Higgs boson as well as the neutral scalar boson are already quite high, it is very unlikely they will cancel the contribution of a relatively light pseudoscalar boson.

In conclusion, in this Rapid Communication we report a set of analytic formulas for the two-loop contributions of a generic pseudoscalar boson to lepton anomalous magnetic moment. Such pseudoscalar bosons may exist in any theory beyond the standard model and they are typically harder to constrain using collider experimental data. In this paper, we show that strong constraint on such sectors can be derived from the precision data on muon anomalous magnetic moments from the going and future experiments. We hope our work adds importance and urgency to these low energy precision experiments.

Note added in proof. After this work was submitted, the E821 experiment announced its updated $g-2$ value, which is well above the SM prediction by 2.6σ. Given this nontrivial result, we have inserted a data bar near the right margin of Fig. 2, indicating the new allowed region of $\Delta a_\mu = (10-75)\times 10^{-10}$ from new physics at 95% C.L. The positive two-loop contribution is able to fit the data, e.g., by large $\tan\beta \sim 50$ and $M_a \lesssim 40$ GeV, as illustrated in Fig. 2. Note that for M_a lighter than roughly 3 GeV, the negative one-loop contribution dominates and gives the overall negative Δa_μ, which is disfavored by the current E821 data.

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